Trigonometric identities

(a)

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

(b)

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(c)

$\sin(90^0 - \theta) = \cos\theta$

$\sin(90^0 + \theta) = \cos\theta$

$\cos(90^0 - \theta) = \sin\theta$

$\cos(90^0 + \theta) = -\sin\theta$

$\tan(90^0 - \theta) = \cot\theta$

$\tan(90^0 + \theta) = -\cot\theta$

(d)

$\sin(180^0 - \theta) = \sin\theta$

$\sin(180^0 + \theta) = -\sin\theta$

$\cos(180^0 - \theta) = -\cos\theta$

$\cos(180^0 + \theta) = -\cos\theta$

$\tan(180^0 - \theta) = -\tan\theta$

$\tan(180^0 + \theta) = \tan\theta$

(e)

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

R-Formulae

(1) $a\sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$

(2) $a\cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$

where $R = \sqrt{a^2 + b^2}$

$$\alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

Addition and Subtraction Formulae

$\sin(A \pm B) =$
$\sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) =$
$\cos A\cos B \mp \sin A\sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double Angle Formulae

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

x	0^0	90^0	180^0	270^0
sin x	0	1	0	-1
cos x	1	0	-1	0
tan x	0	∞	0	∞

x	30^0	45^0	60^0
sin x	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos x	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan x	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

A **stationary point** on a curve is defined as a point where the $\frac{dy}{dx} = 0$.

Test for a stationary pt,
(DT: derivative test)

Max pt:
1st DT: grad. changes from +ve to 0 to -ve.
2nd DT: $\frac{d^2y}{dx^2} < 0$

Min pt:
1st DT: grad. changes from -ve to 0 to +ve.
2nd DT: $\frac{d^2y}{dx^2} > 0$

Pt of inflexion:
1st DT: grad. does not change sign.
2nd DT: $\frac{d^2y}{dx^2} = 0$
Use 1st derivative to test

Derivatives

1. If $y = k$, where k is a const $\frac{dy}{dx} = 0$

2. If $y = kx^n$, $\frac{dy}{dx} = knx^{n-1}$

3. (Chain Rule)
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

4. (Product Rule)
$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

5. (Quotient Rule)
$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

6.Diff. of trigo functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}[\sin(ax+b)] = a\cos(ax+b)$$

$$\frac{d}{dx}[\cos(ax+b)] = -a\sin(ax+b)$$

$$\frac{d}{dx}[\tan(ax+b)] = a\sec^2(ax+b)$$

$$\frac{d}{dx}(\sin^n x) = n\sin^{n-1} x\cos x$$

$$\frac{d}{dx}(\cos^n x) = -n\cos^{n-1} x\sin x$$

$$\frac{d}{dx}(\tan^n x) = n\tan^{n-1} x\sec^2 x$$

7.Diff. of log functions

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}[\ln(f(x))] = \frac{f'(x)}{f(x)}$$

8.Diff. of exp functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

Integrals (c is a constant)

1. Integral of

$$\int(ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$
$(n \neq -1)$

$$\int(ax+b)^n\,dx = \frac{\ln(ax+b)}{a} + c$$
$(n = -1)$

2.Int. of trigo functions

$\int\sin x\,dx = -\cos x + c$

$\int\cos x\,dx = \sin x + c$

$\int\tan x\,dx = -\ln\cos x + c$

$\int\sec^2 x\,dx = \tan x + c$

$$\int\sin ax\,dx = -\frac{\cos ax}{a} + c$$

$$\int\cos ax\,dx = \frac{\sin ax}{a} + c$$

$$\int\sec^2 ax\,dx = \frac{\tan ax}{a} + c$$

$$\int\sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + c$$

$$\int\cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + c$$

$$\int\tan ax\,dx = \frac{-\ln\cos ax}{a} + c$$

$$\int\tan(ax+b)\,dx = \frac{-\ln\cos(ax+b)}{a} + c$$

$$\int\sec^2(ax+b)\,dx = \frac{\tan(ax+b)}{a} + c$$

3. Int. of exp functions

$$\int e^x\,dx = e^x + c$$

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c$$

Area under the graph

1. $\int_a^b y\,dx$
(bounded by x-axis, x=a to x=b)

2. $\int_c^d x\,dy$
(bounded by y-axis, y=c to y=d)

Displacement, velocity, acc

Velocity, $v = \frac{ds}{dt}$

Acceleration, $a = \frac{dv}{dt}$

$s = \int v\,dt$.

$v = \int a\,dt$.